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Power Analysis and Efficiency Calculation of the Complex and Closed Planetary Gears Transmission

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Abstract

2K-H planetary gears transmission power diagram is drawn by analyzing the kinematic, torque and power balance of the basic gear transmission, the corresponding relationship between typical 2K-H planetary gears transmission diagram and power diagram and power balance. Combining one specific example, the power distributions of the complex and closed planetary gears transmission are analyzed and its overall transmission efficiencies is calculated under considering power losses or not. The results show that the graphical representation is a simple, intuitive and practical method for power analysis and efficiency calculation, it can provide theoretical references for design work of the complex and closed planetary gears transmission.

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Keywords: basic circuit; closed planetary gear transmission; power analysis; transmission efficiency;

1. Introduction

Closed planetary gear has the advantages of compact structure, high transmission ratio and small size and so on, it is widely used in modern mechanical transmission [1], but the power analysis and efficiency calculation of the closed planetary gear become difficult to carry out as its complex structure. Pennestrì and Valentini [2] analyzed the planetary gears transmission efficiency of two degrees of freedom, then with Mariti and Del Pio [3, 4] applied graph method to analyze the motion, power flow and transmission efficiency of spur and bevel gear planetary gears transmission. Del Castillo [5] proposed the transmission efficiency analytical expressions for any spur planetary gears transmission by the concepts of speed, torque, virtual gear ratio and the relationship between the power and the

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speed ratio of the gears, then with Salgado [6,7] analyzed the power flow and transmission efficiency of planetary gears transmission with multi-members based on graphical representation method. Laus and Simas, et al, [8] analyzed the transmission efficiency of duplex planetary gears transmission by graph and screw theory. Chen Xiaoan and Chen Hong [9] proposed an analysis geometric method applied to the kinematic and efficiency calculation of compound planetary gear, and then Li Yunsong [10] introduced that method to analyze the complex and closed planetary gears transmission with them. C. Chen [11] analyzed the split-power and transmission efficiency based on the theory of virtual split-power and applied the new concept of the split-power ratio and virtual split-power. Wang Huiwu, et al, [12] analyzed the characteristics of the power flow, transmission efficiency and self-lock conditions, and proposed a new analysis method of critical power design parameters which is the ratio of the main branch power to the input total power. Dong Wanfu, et al, [13] analyzed the power flow based on the relationship diagram between the structure and power flow of closed planetary gear train. Li Qingkai, et al, [14] applied bond graph theory to analyze the power flow of the closed planetary gear train.

The methods reported were cumbersome to analyze power flow of the complex and closed planetary gears transmission and calculate its efficiency. They were also difficult for practical application, therefore much of difficulties for design work of the complex and closed planetary gears transmission were caused and the development and applications were restricted seriously. The power distributions of the complex and closed planetary gears transmissions of the aircraft engine power split planetary gear reducer is analyzed and its transmission efficiency is calculated based on two rules, which are all types of complex or compound gear can be decomposed into basic gear transmission with a single degree of freedom or two degrees of freedom and the basic gear transmission meets with the torque and power balance. The results show that the graphical representation is a simple, intuitive and practical method for power analysis and efficiency calculation, it can provide theoretical references for design work of the complex and closed planetary gears transmission.

2. Basic gear transmission (BGT) analysis[3,15]

2.1. Structure Kinematic and power analysis of BGT

All types of complex or compound gear can be decomposed into basic gear transmission with a single degree of freedom or two degrees of freedom. The kinematic relation of BGT is shown in Fig. 1, where *i*, *j* represent the gear, *k* represents the gear frame, *G* represents the gear meshing pair, *R* (a) and *R* (b) represent the rotating pairs of the gear *i*, *j* with the gear frame *k*.

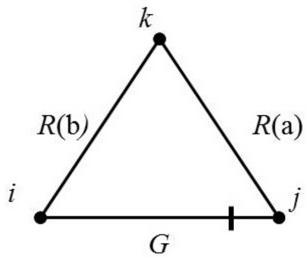


Fig. 1. Kinematic relations of the gear *i*, gear *j* and gear frame *k*

The number of teeth ratio of the gear *i* and the gear *j* is shown as formula (1).

$$R_m = \pm \frac{z_{jm}}{z_{im}} \tag{1}$$

where R_m is the gear ratio of the gear *i* and the gear *j* in the m^{th} circuit, z_{im} and z_{jm} are number of teeth of the gear *i* and the gear *j*,where the ± sign represents internal and external gear pairs, respectively.

The kinematic equation of the gear *i*, the gear *j* and the gear frame *k* is shown as formula (2).

$$\omega_i - R_m\omega_j + \left(R_m - 1\right)\omega_k = 0 \tag{2}$$

where ω_i, ω_j and ω_k are the angular velocity of the gear *i*, the gear *j* and the gear frame *k*.

The torque and power balance equation of the gear *i*,the gear *j* and the gear frame *k* are shown as formula (3), (4).

$$T_{im} + T_{jm} + T_{km} = 0 \tag{3}$$

$$P_{im} + P_{jm} + P_{km} = 0 \tag{4}$$

where T_{im}, T_{jm} and T_{km} are the torque of the gear *i*, the gear *j* and the gear frame *k*, P_{im}, P_{jm} and P_{km} are the power of the gear *i*, the gear *j* and the gear frame *k*.

2.2. Efficiency analysis of BGT

The transmission relation diagram of BGT can be drawn by the following method. The gear *i*, the gear j and the gear frame k are shown by node • and their numbers, the basic circuit composed of the gear *i*, the gear *j* and the gear frame *k* is shown by circle ○ and their numbers. The sign ⏚ and its number indicate one of the gear *i*, the gear *j* or the gear frame *k* is fixed. All basic circuit and nodes meet with the power balance equation of the gear *i*, the gear *j* and the gear frame *k*.

The transmission relation diagram of BGT with single degree of freedom (SDOF) only has 6 basic types and their efficiency expression are shown in Table 1 or in the derivation process of the literature[3]. In the Table 1, $\left(\eta_{ij}^{0}\right)^{-1} = 1/\eta_{ij}^{0}$, where η_{ij}^{0} is basic meshing efficiency between the gear *i* and the gear *j* and it can be accurately calculated by the formula (5) [16].

$$\eta_{ij}^{0} = 1 - \pi\mu\left(\frac{1}{z_i} \pm \frac{1}{z_j}\right)\left(\varepsilon_1^2 + \varepsilon_2^2 - \varepsilon_1 - \varepsilon_2 + 1\right) \tag{5}$$

This formula is used in the contact ratio range $1 < \varepsilon_\alpha < 2$, the friction coefficient μ is constant, z_i is the teeth number of the gear *i*, z_j is the teeth number of the gear *j*. $\varepsilon_1 = g_1/p_b$, $\varepsilon_2 = g_2/p_b$, $\varepsilon_\alpha = \varepsilon_1 + \varepsilon_2$, g_1 is from pitch point *p* to meshing finish point B_1, g_2 is from meshing starting point B_2 to pitch point *p*, p_b is base pitch, in this paper η_{ij}^{0}=0.980[16].

The transmission relation diagram of BGT with two degree of freedom are also only 6 basic types. The transmission efficiency expression of the basic circuit of the following example of single-input dual-output (SIDO) will be derived. The gear *i* as input, the gear *j* and the gear frame *k* as outputs (see Fig. 2), their basic circuits meet with the formula (2) and (3). During deriving the transmission efficiency calculation formula of the SIDO basic circuit, it is assumed that the gear *j* or the gear frame *k* is fixed every time, and then combine their power balance equation, so the transmission efficiency calculation formula of the SIDO basic circuit shown in Fig. 2 can be derived.

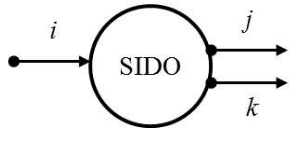


Fig. 2. SIDO basic circuit transmission diagram

Firstly, assuming the gear j is fixed, the efficiency $\eta_{j(i-k)}$ can be calculated by the formula(6).

$$\eta_{j(i-k)} = -\frac{T_k \omega_k}{T_i \omega_i} \tag{6}$$

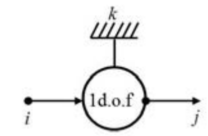
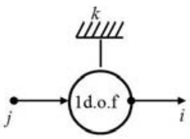
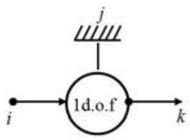
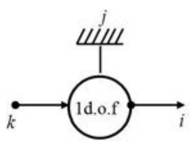
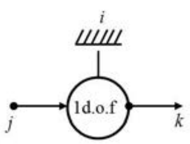
Then, assuming the gear frame k is fixed, the efficiency $\eta_{k(i-j)}$ can be calculated by the formula(7).

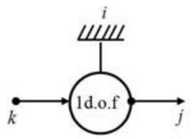
$$\eta_{k(i-j)} = -\frac{T_j \omega_j}{T_i \omega_i} \tag{7}$$

When assuming the gear j is fixed, $\omega_j = 0$, ω_k can be obtained by the formula (2) and shown by formula (8) below.

$$\omega_k = \frac{1}{1 - R_m} \omega_i \tag{8}$$

Table 1. SDOF basic gear transmission diagram and efficiency

Type	Transmission diagram	R_m range	Transmission efficiency
1			η_{ij}^{0}
2			$\left(\eta_{ij}^{0}\right)^{-1}$
3		$R_m<0$ or $R_m>1$	$\frac{R_m \eta_{ij}^{0} - 1}{R_m - 1}$
		$0<R_m<1$	$\frac{R_m \left(\eta_{ij}^{0}\right)^{-1} - 1}{R_m - 1}$
4		$R_m<0$ or $R_m>1$	$\frac{R_m - 1}{R_m \left(\eta_{ij}^{0}\right)^{-1} - 1}$
		$0<R_m<1$	$\frac{R_m - 1}{R_m \eta_{ij}^{0} - 1}$
5		$R_m<1$	$\frac{R_m - \eta_{ij}^{0}}{R_m - 1}$
		$R_m>1$	$\frac{R_m - \left(\eta_{ij}^{0}\right)^{-1}}{R_m - 1}$

6		$R_m<1$ $R_m>1$	$\frac{R_m-1}{R_m-\left(\eta_{ij}^{0}\right)^{-1}}$ $\frac{R_m-1}{R_m-\eta_{ij}^{0}}$

When assuming the gear frame k is fixed, $\omega_k=0$, ω_j can be obtained by the formula (2) and shown by formula (9) below.

$$\omega_j=\frac{1}{R_m}\omega_i \tag{9}$$

Taking the equation (8) into the formula (6) yields have T_k which is shown by the formula (10) below.

$$T_k=T_i\eta_{j(i-k)}\left(R_m-1\right) \tag{10}$$

Taking the equation (9) into the formula (7) yields T_j which is shown by the formula (11) below.

$$T_j=-T_i\eta_{k(i-j)}R_m \tag{11}$$

$$\eta_E T_i\omega_i+T_j\omega_j+T_k\omega_k=0 \tag{12}$$

Taking the equation (8), (9), (10) and (11) into the formula (12) yields η_E which is shown by the formula (13) below.

$$\eta_E=\frac{\eta_{j(i-k)}\left(1-R_m\right)\omega_k+\eta_{k(i-j)}R_m\omega_j}{\omega_i} \tag{13}$$

3. Power analysis and efficiency calculation of the complex and closed planetary gears transmission

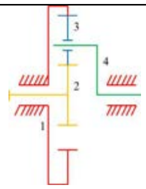
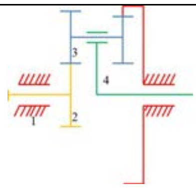
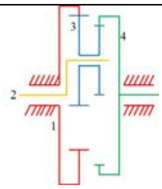
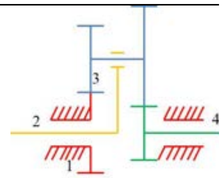
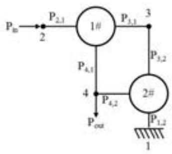
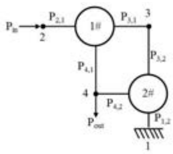
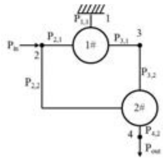
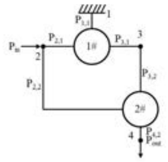
3.1. 2K-H planetary gears transmission power diagram and power balance equations

2K-H planetary gears transmission power diagram is graphical representation method that labels power symbols on each basic circuit of planetary gears transmission which is composed of BGT. All basic circuits and nodes of 2K-H planetary gears transmission power diagram meet with the power balance equation. 2K-H planetary gears transmission power diagram and the power balance equations are shown in Table 2.

From table 2, it can be found that the nature of power diagram and power balance equations between NGW type and NW type planetary gears transmission are the same, and the nature of power diagram and power balance equations between NN type and WW type planetary gears transmission are also the same.

Table 2. 2K-H planetary gears transmission power diagram and balance equations

Transmission type	NGW	NW	NN	WW

Structure diagram				
Power diagram				
Power balance equations	$P_{2,1}+P_{3,1}+P_{4,1}=0$ $P_{3,2}+P_{4,2}+P_{1,2}=0$ $P_{in}+P_{2,1}=0$ $P_{3,1}+P_{3,2}=0$ $P_{out}+P_{4,1}+P_{4,2}=0$ $P_{1,2}=0$	$P_{2,1}+P_{3,1}+P_{4,1}=0$ $P_{3,2}+P_{4,2}+P_{1,2}=0$ $P_{in}+P_{2,1}=0$ $P_{3,1}+P_{3,2}=0$ $P_{out}+P_{4,1}+P_{4,2}=0$ $P_{1,2}=0$	$P_{1,1}+P_{2,1}+P_{3,1}=0$ $P_{2,2}+P_{3,2}+P_{4,2}=0$ $P_{in}+P_{2,1}+P_{2,2}=0$ $P_{3,1}+P_{3,2}=0$ $P_{out}+P_{4,2}=0$ $P_{1,1}=0$	$P_{1,1}+P_{2,1}+P_{3,1}=0$ $P_{2,2}+P_{3,2}+P_{4,2}=0$ $P_{in}+P_{2,1}+P_{2,2}=0$ $P_{3,1}+P_{3,2}=0$ $P_{out}+P_{4,2}=0$ $P_{1,1}=0$

3.2. Application Example

One example is taken in the literature[17]as application of the above method. It is known that an aircraft engine power split planetary gear reducer(AEPR) diagram shown in Fig. 3, its numbers of teeth are $z_2 = z_{4'} = 35$, $z_3 = z_5 = 31$, $z_4 = z_6 = 97$, its input angular velocity $\omega_2 = 12300$r/min.

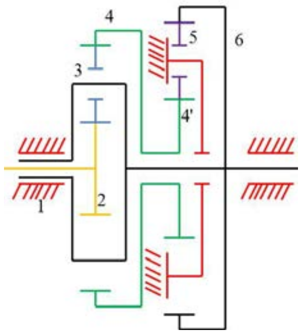


Fig. 3. Aircraft engine power split planetary gear reducer diagram

Kinematic analysis on AEPR shown in Fig. 3 produces the angular velocity of the transmission body and each gear in Table 3.

Table 3. The angular velocity of the aircraft engine power split planetary gear reducer (r/min)

ω_1	ω_2	ω_3	ω_4	ω_5	ω_6
0	12300	-11600.17	-2976.48	3360.60	1074

Transmission relations of AEPR shown in Fig. 3 can be seen in Table 4. Power diagram of AEPR is shown in Fig. 4. Power analysis on AEPR are discussed in two different cases as follows.

Table 4. Transmission relations of the aircraft engine power split planetary gear reducer

Basic circuit	*i*	*j*	*k*	R_m
1#	2	3	6	-0.886
2#	3	4	6	3.129
3#	4	5	1	-0.886

4#	5	6	1	3.129

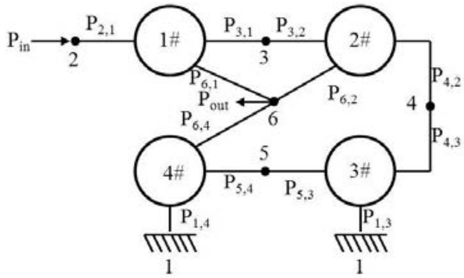


Fig. 4. The power diagram of the aircraft engine power split planetary gear reducer

Case Ⅰ: without considering power losses

There are 4 basic circuits and 5 nodes in Fig. 4. All basic circuits and nodes meet with the power balance equations. The power balance equations of the basic circuits are shown from the equation (14) to (17). The power balance equations of the nodes are shown from the equation (18) to (22).

$$1\#\ \ P_{2,1}+P_{3,1}+P_{4,1}=0 \tag{14}$$

$$2\#\ \ P_{3,2}+P_{4,2}+P_{6,2}=0 \tag{15}$$

$$3\#\ \ P_{4,3}+P_{5,3}+P_{1,3}=0 \tag{16}$$

$$4\#\ \ P_{5,4}+P_{6,4}+P_{1,4}=0 \tag{17}$$

$$2\#\text{ node: } -P_{in}+P_{2,1}=0 \tag{18}$$

$$3\#\text{ node: } P_{3,1}+P_{3,2}=0 \tag{19}$$

$$4\#\text{ node: } P_{4,2}+P_{4,3}=0 \tag{20}$$

$$5\#\text{ node: } P_{5,4}+P_{5,3}=0 \tag{21}$$

$$6\#\text{ node: } P_{out}+P_{6,1}+P_{6,2}+P_{6,4}=0 \tag{22}$$

There is not power flow through the fixed transmission body, therefore the following equations should be applied.

$$P_{1,3}=0 \tag{23}$$

$$P_{1,4}=0 \tag{24}$$

Assuming P_{in} = 1, without considering the power losses, the power values of AEPR in Fig. 4 can be obtained which are shown in Table 5, the power distributions can be seen in Fig. 5 (a).

Case Ⅱ: considering power losses

1#, 2# basic circuits are two degrees of freedom BGT and 3#, 4# basic circuits are single degree of freedom BGT, Appling the method shown in 2.2 to calculate the transmission efficiency of each basic circuit.

1# basic circuit transmission diagram shown in Fig. 6(a), R_1=-0.886, one can obtain the transmission efficiency of the 1# basic circuit through the following three steps. Firstly, assuming 3(j) is fixed, one can obtain $\eta_{3(2\text{-}6)}$ = 0.991. Next, assuming 6(k) is fixed, similarly yields $\eta_{6(2\text{-}3)}$ = 0.991. Finally, summing the transmission efficiency of previous two steps to calculate the transmission efficiency of the 1# basic circuit, η_{E1} = 0.982 can be obtained, that is the equations (25), (26) and (27).

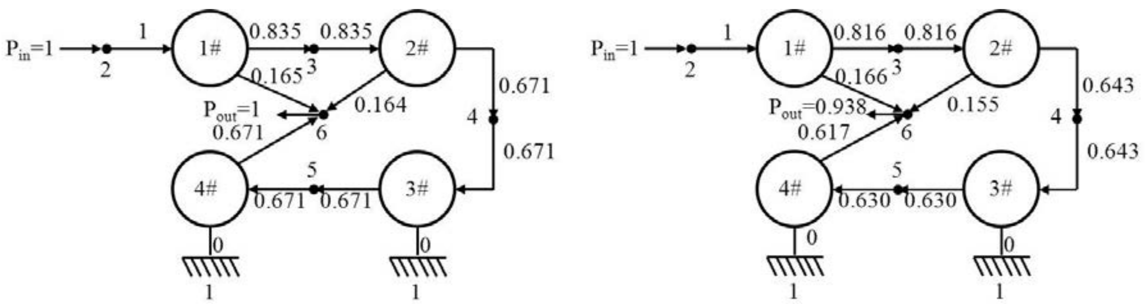
Table 5. The power value of the aircraft engine power split planetary gear reducer

Without considering power losses		Considering power losses	
$P_{2,1}$=1	$P_{3,1}$=-0.835	$P_{2,1}$=1	$P_{3,1}$=-0.816
$P_{6,1}$=-0.165	$P_{3,2}$=0.835	$P_{6,1}$=-0.166	$P_{3,2}$=0.816
$P_{4,2}$=-0.671	$P_{6,2}$=-0.164	$P_{4,2}$=-0.643	$P_{6,2}$=-0.155
$P_{4,3}$=0.671	$P_{5,3}$=-0.671	$P_{4,3}$=0.643	$P_{5,3}$=-0.630
$P_{5,4}$=0.671	$P_{6,4}$=-0.671	$P_{5,4}$=0.630	$P_{6,4}$=-0.617
$P_{1,4}$=0	$P_{1,3}$=0	$P_{1,4}$=0	$P_{1,3}$=0
P_{in}=1	P_{out}=1	P_{in}=1	P_{out}=0.938

$$\eta_{3(2-6)} = \frac{R_m \eta_{ij}^0 - 1}{R_m - 1} = 0.991 \tag{25}$$

$$\eta_{6(2-3)} = \eta_{ij}^0 = 0.980 \tag{26}$$

$$\eta_{E1} = \frac{\eta_{j(i-k)}\left(1-R_m\right)\omega_k + \eta_{k(i-j)} R_m \omega_j}{\omega_i} = 0.982 \tag{27}$$



(a) (b)

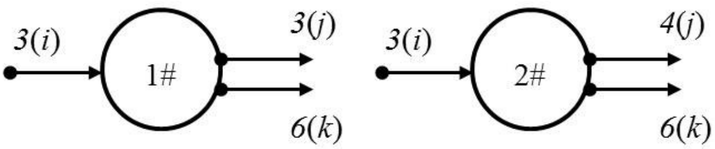
Fig. 5. The power distributions of the aircraft engine power split planetary gear reducer

2# basic circuit transmission diagram shown in Fig. 6(b), R_2=3.129, $\eta_{4(3\text{-}6)}$=0.971, $\eta_{6(3\text{-}4)}$=0.980, η_{E2}=0.978 were obtained respectively through the calculation process which is the same as 1# basic circuit transmission as the following equations (28), (29) and (30).

$$\eta_{4(3-6)} = \frac{R_m \eta_{ij}^0 - 1}{R_m - 1} = 0.971 \tag{28}$$

$$\eta_{6(3-4)} = \eta_{ij}^0 = 0.980 \tag{29}$$

$$\eta_{E2}=\frac{\eta_{j(i-k)}\left(1-R_m\right)\omega_k+\eta_{k(i-j)}R_m\omega_j}{\omega_i}=0.978 \tag{30}$$



(a) (b)

Fig. 6. 1#, 2# basic circuit transmission diagram

3# basic circuit transmission diagram shown in Fig. 7(a), R_3=-0.886, the transmission efficiency η_{E3} can be directly calculated by the formula shown in Table 1, see equation (31).

$$\eta_{E3}=\eta_{ij}^{0}=0.980 \tag{31}$$

4# basic circuit transmission diagram shown in Fig. 7(b), R_4=3.129, the transmission efficiency η_{E4} can also be directly calculated by the formula shown in Table 1, see the equations (32).

$$\eta_{E4}=\eta_{ij}^{0}=0.980 \tag{32}$$

Considering power losses, all basic circuits and nodes of AEPR also meet with the power balance equations, the power balance equations of the basic circuits are shown from the equation (33) to (36). The power balance equations of the nodes are shown from the equation (37) to (41).

1# $0.982P_{2,1}+P_{3,1}+P_{4,1}=0$ (33)

2# $0.978P_{3,2}+P_{4,2}+P_{6,2}=0$ (34)

3# $0.980P_{4,3}+P_{5,3}+P_{1,3}=0$ (35)

4# $0.980P_{5,4}+P_{6,4}+P_{1,4}=0$ (36)

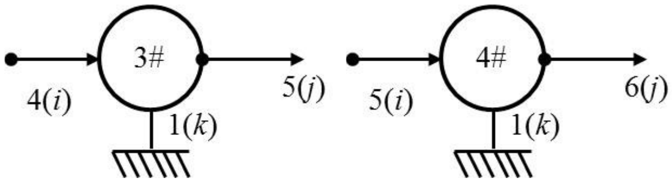
2# node: $-P_{in}+P_{2,1}=0$ (37)

3# node: $P_{3,1}+P_{3,2}=0$ (38)

4# node: $P_{4,2}+P_{4,3}=0$ (39)

5# node: $P_{5,4}+P_{5,3}=0$ (40)

6# node: $P_{out}+P_{6,1}+P_{6,2}+P_{6,4}=0$ (41)



(a) (b)

Fig. 7. 3#, 4# basic circuit transmission diagram

There is not power flow through the fixed transmission body, therefore the following equations should be applied.

$$P_{1,3}=0 \tag{42}$$

$$P_{1,4}=0 \tag{43}$$

Combining the equations (2), (3) and (12), the relationship equation P_{jm} and P_{im} can be obtained from equation (44).

$$\frac{P_{jm}}{P_{im}}=-\frac{\left(\omega_i-R_m\omega_j\right)\omega_j+\eta_E\left(R_m-1\right)\omega_i\omega_j}{\left(\omega_i-\omega_j\right)\omega_i} \tag{44}$$

The relationship equation P_{jm} and P_{im} of the other 5 types of basic circuits, reference to the literature [3].

Combining the characteristics of 1#, 2# basic circuits and the equation (44), the equations (45) and (46) can be obtained.

$$\frac{P_{3,1}}{P_{2,1}}=-0.816 \tag{45}$$

$$\frac{P_{4,2}}{P_{3,2}}=-0.788 \tag{46}$$

Combining the equations (33) to (46) and assuming P_{in}=1, when considering the power loss, the power values of AEPR in Fig. 4 can be obtained which are shown in Table 5, the power distributions can be seen in Fig. 5 (b).

$$\eta=\left|\frac{P_{out}}{P_{in}}\right|=0.938 \tag{47}$$

The overall transmission efficiency of AEPR in Fig. 3 is 0.938 by the calculation of the equation (47).

4. Conclusions

Anlalyzing power and calculating the transmission efficiency of the complex and closed planetary gears transmission in this paper based on the kinematic equation, torque and power balance equation of the basic gear transmission. The calculation results of the transmission efficiency by the method of the paper is consistent with the conventional method, but this method is more simple, intuitive and practical method for power analysis and efficiency calculation, it can provide theoretical references for design work of the complex and closed planetary gears transmission.

The method in this paper can achieve power analysis and efficiency calculation of more complex compound planetary gears transmission through the analytical processes of Example.

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